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TRANSPORTATION RESEARCH COMMAND
FORT EUSTIS, VIRGINIA

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TRECOM TECHNICAL REPORT 63-67H

MISSION SURVIVABILITY OF A MANNED AIRCRAFT SURVEILLANCE SYSTEM

A MATHEMATICAL MODEL FOR THE VISUAL DETECTION OF LOW-FLYING AIRCRAFT

Volume VIII

Task 9R38-10-005-01
Contract DA 44-177-TC-793

December 1963

prepared by:

CANADAIR Ltd.
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MISSION SURVIVABILITY OF A MANNED AIRCRAFT SURVEILLANCE SYSTEM

VOLUME VIII

A MATHEMATICAL MODEL FOR THE VISUAL DETECTION OF LOW-FLYING AIRCRAFT

(Canadair Report ASP-1037, February 1963)

(10) by Nicholas Hardyla

Prepared by
Canadair Ltd.
Montreal, Canada

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for

U.S. ARMY TRANSPORTATION RESEARCH COMMAND
FORT EUSTIS, VIRGINIA

PREFACE

In this volume the problem of visual detection of low-flying aircraft is considered as part of a general investigation into the survivability of surveillance aircraft penetrating enemy territory at low altitude.

The work reported upon herein has been sponsored by the U.S. Army Transportation Research Command (USATRECOM) under Task Number 9R 38-10-005-01. It was conducted at Canadair Ltd., under Contract Number DA 44-177-TC-793 with Col. W.F. Molesky, USATRECOM, acting as Project Officer. The principal contributor to this phase of the study was Mr. N. Kurdyla.

CONTENTS

ILLUSTRATIONS

SYMBOLS

Symbol	
A_b	is the bottom projected area of target in square feet
A_f	is the frontal projected area of target in square feet
A_p	is the presented area of target in square feet
A_s	is the side projected area of target in square feet
C	is the contrast of the target relative to the background at the lens of the eye in percent
C_1	is the resultant contrast which takes into account the effective contrast and the variation in threshold contrast of an individual in percent
$C_{eff}=(\Delta I/I)_{eff}$	is the effective contrast when light quanta falls within the integration time and integration distance of the retina of the eye in percent
C_o	is the intrinsic contrast between target and background in percent
$C_t=(\Delta I/I)_t$	is the threshold contrast of an individual's eye in percent
d	is the instantaneous minimum distance from the target to the line of sight in miles
d_{ij}	is the minimum distance from the target to the line of sight of the i^{th} glimpse and j^{th} flight path in miles
d'_{ij}	is the minimum distance from the target to the line of sight at the beginning of the i^{th} scan period during j^{th} flight path in miles
d_{ijl}	is the minimum distance of target's position to the line of fixation when it intercepts the lobe boundary (50 percent lobe) in miles

SYMBOLS

Symbol	
d'_{ij_1}	is the minimum distance of target's position to the line of fixation when it leaves the bounds of the detection lobe in miles
D_{ij}	is the total distance the target traverses while within the detection lobe in miles
h	is the height of the target above ground level in feet
I	is the background light intensity in Lamberts
I'	is the light intensity when a target obscures part of the background in Lamberts
ΔI	is the difference in light intensity reaching the eye when only the background light falls on the eye, as compared to the background light falling onto the eye with target present in Lamberts
$\Delta I'$	is the effective differential intensity of a flashing light in Lamberts
K	is the number of sections around the retinal image perimeter
M	is the total number of quanta impinging on the retina
M_{eff}	is the number of light quanta impinging on the retina that fall within the integration capability of the retina (when source of quanta is moving)
n	is the number of light quanta above the background intensity required for the perception of light if light quanta are absorbed within δ and τ_e of each other
N_k	is the number of points in the k^{th} section of histogram

SYMBOLS

Symbol	
p	is the probability of absorption of a quanta per trial
$P_d = P(D/L_s)$	is the probability of visual detection given line of sight to the target
P_{d_k}	is the probability of detection for the k^{th} interval in the histogram
P_n	is the probability that at least n light quanta will be absorbed in a given region of the boundary
P_{see}	is the probability that a nerve pulse will be triggered by light quanta falling on the retina
$P_{see\,ij}$	is the probability that a nerve pulse will be triggered by light quanta falling on the retina for i^{th} glimpse or scan and j^{th} flight path
$P_{see_{k\ell}}$	is the probability of seeing for the ℓ^{th} point in the k^{th} section of the histogram
$P(\vartheta, R)$	is the probability of detection as a function of range and angular position of the aircraft relative to the line of sight
R	is the range in appropriate units
R_1	is the range upon entering the detection lobe in miles
R_1'	is the range upon leaving the detection lobe in miles
R_m	is the maximum detection range in miles
t_{ij}	is the time coordinate at the commencement of i^{th} glimpse during j^{th} flight path in seconds
t'_{ij}	is the time coordinate at the commencement of i^{th} scan during j^{th} flight path in seconds

SYMBOLS

Symbol	
$t_{i+1,j}$	is the time coordinate at the commencement of i^{th} glimpse and j^{th} flight path in seconds
T_f	is the time period allotted for the glimpse action of the search for all glimpses in seconds
T_{fij}	is the time during which target is effectively within the perceptibility region for i^{th} glance and j^{th} flight path in seconds
T_m	is the time period allotted for the scanning action of the search for all scans in seconds
v	is the target velocity along a straight line flight path in feet per second
v_x, v_y	are the velocities along the X axis and the Y axis in feet per second
V	is the meteorological visibility in miles
x_{oj}	is the distance from the observer measured along the X axis at which the flight path intercepts the X axis in feet
X, Y	are the instantaneous coordinates of the target in miles
X_{ij}, Y_{ij}	are the coordinates of the target when the observer is commencing the i^{th} glimpse during the j^{th} flight path in miles
X_{i+1j}, $Y_{i+1,j}$	are the position coordinates of the target at the end of the scan period or alternatively the beginning of the i^{th} glimpse j^{th} flight path in miles

SYMBOLS

Symbol	
X'_{ij}, Y'_{ij}	are the coordinates of target at the beginning of scanning phase of the search in miles
δ	is the angular distance on the retina within which the integration probability of the eye approaches one in minutes.
ϵ	is the expected number of successes, or absorptions of quanta by the retina
ϵ'	is the expected average number of absorptions of light quanta from moving targets falling on the retina
ϵ_{eff}	is the expected average number of absorptions of light quanta (from a moving target) which fall within the integrating capacity of the retina
θ	is the instantaneous angular orientation of the observer in degrees
$d\theta/dt$	is the rate of angular scan by the observer in minutes per second
θ_{oj}	is the angular orientation of the observer when target first comes within visual range
θ_{ij}	is angular orientation of observer during the i^{th} glimpse j^{th} flight path in degrees
ξ	is the off-line of sight axis visual angle in degrees
ϑ	is the instantaneous angular position of the target relative to the observer in minutes
$d\vartheta/dt$	is the retinal angular velocity of the targets image in minutes per second

SYMBOLS

Symbol	
τ	is the time of exposure for visual detection in seconds
τ_e	is the time constant of the eye within which time the integration probability of the retina approaches one in seconds
τ_f	is the duration of the on-phase of the flashing signal or alternatively the duration that the aircraft remains within the perceptibility region in seconds
ϕ_{oj}	is an angle defining the slope of the j^{th} flight path in degrees
χ	is a uniformly random variable defined between 0 and 1

SUMMARY

This volume presents the derivation of a mathematical model for use in the determination of probability of visual detection of low-flying aircraft. The result is the factor $P(D/L_s)$, probability of visual detection given line of sight, which can be used in the general investigation of survivability of surveillance aircraft against visually aimed weapons.

The mathematical formulation of the situation is based on geometrical concepts of a two-dimensional field, the known characteristics of the human eye, and an assumed technique of scanning by an observer.

To determine the probability of visual detection in a given situation, a Monte Carlo approach, utilizing a digital computer, is proposed; the results, taken over a large number of random flight paths, would yield a probability of detection as a function of initially chosen parameters.

No numerical results are presented in this report because of the unavailability to the contractor of a digital computer of sufficient capacity to make such solutions obtainable in an economical and efficient manner.

An extensive literature survey was carried out in conjunction with this study. Although only References 1 through 6 were used directly in this volume, the others listed are closely connected with the visual detection subject and could prove useful in further studies.

CONCLUSIONS

The problem of visual detection of a rapidly moving target, although essentially complex, is amenable to analysis of the Monte Carlo type. The mathematical formulation in the present report is offered as an approach which considers all of the apparently significant parameters relating to this problem.

RECOMMENDATIONS

The mathematical model given in this report has not yet been numerically evaluated since limitations to contractor's computing equipment have precluded an economical solution.

To establish the validity of the mathematical model, it is recommended that a Monte Carlo process be carried out on a high-speed computer to obtain the probability of detection as a function of range for various parameters such as target size, target contrast, meteorological visibility, crossing distance and target velocity for a given search pattern.

Some experimental results are available in Reference 1. The model should be checked using parameters for which experimental results are available.

INTRODUCTION

Among the problems related to the study of survivability of low-flying high-speed surveillance aircraft is the determination of the probability of visual detection given line of sight $P(D/L_s)$ by ground observers. The most general approach to this problem would involve the study of the interrelation of terrain effects to the dynamics of the observer-target system. This approach would not, however, lead to a general solution but would depend upon the characteristics of the terrain in which the observer-target system were located. Before this problem can be solved, it is essential that the characteristics of the observer-target system in an idealized situation be understood. The present report is an attempt to provide such an understanding for an idealized situation where no obstructions exist; the earth is assumed to be flat, and the line of sight distance is given in plan only.

A mathematical formulation based upon the geometry of the engagement situation, the known characteristics of the human eye, and an assumed scan procedure of scan-fixate-scan is presented.

A solution by the use of Monte Carlo techniques for the probability density distribution of sightings, within the mathematical framework presented, is proposed.

ASSUMPTIONS

In this study, the problem of visual detection is only taken to a point where the increase or decrease of light energy due to a target in the field of view causes a perceptable alteration in the rate of triggering of nerve impulses sent to the brain. The problems of noticing these changes once they arrive in the brain are not considered. Thus, the vigilance behavior of the observer is not taken into account, and he is always assumed to be in the optimum state of awareness and readiness.

The assumptions within the limits of the study are given below:

1) Visual detection probabilities are required on the assumption that no obstructions to vision occur except those directly related to light extinction by the atmosphere. The earth is assumed to be flat as far as the eye can see.

2) Within the range of visual bounds of detection as determined by atmospheric conditions at a given time, the aircraft is assumed to follow straight-line paths.

3) Since only low-level high-speed missions are being considered, the aircraft spend a large fraction of the time very close to the horizon altitude. Therefore, a two-dimensional approach to the detection problem seems justifiable for a good first-order approximation.

4) To simulate a possible field situation, the aircraft is assumed to enter the visual detection area in a random fashion both as to orientation and as to crossing distance.

5) The observer is assumed to be searching in a scan-fixate-scan fashion, a fixed time being given for each operation. Different functional behaviors can be given for the scan-fixate portion of the search.

6) An aircraft is assumed to be a point source in relation to the geometric situation. Where the area plays an important part in empirical equation, it has been included. No matter what the orientation of the aircraft relative to the observer, the linear dimension of the target will be obtained by assuming that the presented area is circular.

7) While the eye is fixated in given direction, the aircraft will be represented by a line element whose length represents the distance traversed by aircraft during the fixated glimpse time.

8) During the scan portion of the search, the angular velocity of the aircraft across the retina will be assumed to be due to the eye's scan rate.

9) Attraction of the observer by acoustical energy is considered negligible since the observer is already assumed to be in state of awareness towards airborne targets, and since acoustical energy is highly nondirectional, it serves only to keep the observers vigilant.

10) The velocity of the aircraft is assumed to be a constant for given flight path.

DISCUSSION OF THE VISUAL DETECTION PROBLEM

An observer, who is visually searching for low-level high-speed aircraft, is located at coordinate position (0,0) as shown in Figure 1. All the observer's scan-fixate angular positions and all aircraft angular positions are measured relative to the observer in a counterclockwise direction with the positive X axis as reference direction.

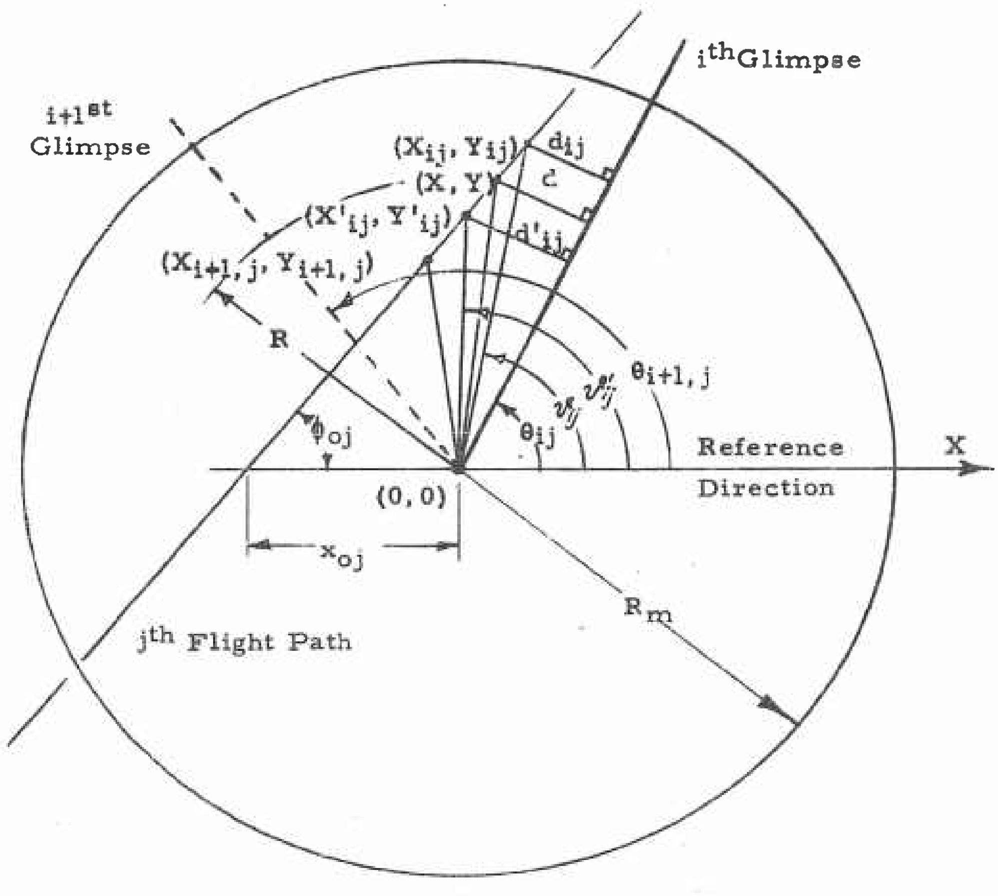

FIGURE 1.

GEOMETRICAL INTERACTION BETWEEN OBSERVER AND TARGET

Since many flight paths need to be considered to obtain a probability density of detection as a function of range, R, and time, τ , let the flight path shown in Figure 1 be the j^{th}. When the aircraft comes within a radius R_m (the visual detection boundary to be defined later) of observer, let the observer be in the process of glancing in a random direction, θ_{oj}. During the progress of the aircraft along its flight path, the observer is simultaneously going through a set search pattern of scan-glimpse actions. Thus, when the aircraft is located at position (X_{ij}, Y_{ij}) at time t_{ij}, an aircraft on its flight path has reached a point in space where the observer is commencing the i^{th} glimpse for the still undetected target. The exact number of glimpses will depend upon the meterological conditions which determine R_m and upon aircraft velocity, v.

By the time the i^{th} glimpse is completed, the aircraft previously at point (X_{ij}, Y_{ij}) has moved at constant velocity to the point (X_{ij}, Y_{ij}). At this time, t_{ij}, the scan operation comes into effect during which time the aircraft moves continuously to a new position. At the end of the scan period the aircraft is located at ($X_{i+1,j}$, $Y_{i+1,j}$), and the time becomes $t_{i+1,j}$, whereupon, a glimpse is taken again.

When the observer takes the i^{th} glimpse for the j^{th} flight path, the angular orientation of observer is denoted by θ_{ij}. For i + 1 st glimpse, angular orientation is denoted by $\theta_{i+1,j}$. Similarly, the aircraft's angular positions relative to the observer are denoted by ϑ_{ij} for the beginning of the i^{th} fixation period, and ϑ'_{ij} for the beginning of the i^{th} scan period. With this notation there is also the freedom of using general variables without subscripts to denote instantaneous positions of glimpses, scans, and aircraft positions as is done later on.

In reality, an aircraft flight path is continuous but in this discrete operation representation of the situation, the flight path must be segmented into line elements representing the sequential relations between observer's search operations and aircraft's time positions as illustrated in Figure 1. The necessity for the separation of events into two discrete cases is necessitated by the characteristic operation of the human eye. Maximum acuity is obtained when the eye fixates a given point of space for a period long compared to a critical length of time, τ_c . However, under the dynamic conditions involved in the present study, the situation in the sky-target position changes appreciably during the time spent in re-fixating. Moreover, the ability of the eye to detect is not lost during the scan portion of the search, but, instead, merely obeys a law strongly dependent on the rate of angular movement of the image on the retina. Thus, it is deemed necessary to consider both effects for an accurate description of the search phases involved in detection. Also, by considering these two effective modes of detection, it may be possible to specify an optimum combination of glimpse-scan times for greatest detecting efficiency.

Denote the j^{th} random flight path by its angular inclination to positive X axis, ϕ_{oj}, and by its crossing point on the X axis, x_{oj}. During the time of the i^{th} fixation, T_f, the aircraft moves a distance vT_f along its flight path from the point (X_{ij}, Y_{ij}) to the point $(X_{ij}-vT_f \cos \phi_{oj}, Y_{ij}-vT_f \sin \phi_{oj})$. Subsequently, when scanning takes place, the aircraft moves along from position $(X_{ij}-vT_f \cos \phi_{oj}, Y_{ij}-vT_f \sin \phi_{oj})$ to the position $[X_{ij}-v(T_f + T_m) \cos \phi_{oj}, Y_{ij}-v(T_f + T_m) \sin \phi_{oj}]$ where T_m is the time period allotted for the i^{th} scanning action.

From the above description, the following relationships follow:

$$
\begin{aligned}
X_{i+1,j} &= X_{ij} - v\,(T_f + T_m) \cos \phi_{oj} \\
Y_{i+1,j} &= Y_{ij} - v\,(T_f + T_m) \sin \phi_{oj}
\end{aligned}
\qquad (1)
$$

During the scan time, T_m, the observer sweeps from a fixation angle θ_{ij}, to the next fixation point $\theta_{i+1,j}$. Several alternatives are available in describing the behavior of a human observer when searching. One is to assume a random glimpse between the boundaries of the search area as has been done in most previous reports on this subject. The second way of obtaining a description of the re-fixation process is to assume that search glimpses occur in a pattern transferring from one to the successive one by means of a random function which weights the glimpses towards smaller values. This method or one similar to it helps simulate the realistic situation where an observer is much more likely to scan areas in the vicinity of previous glance rather than 'look' at widely displaced angles, unless other attention-getting information is present. Therefore, glances should be weighted towards smaller angular displacements and this will be done as indicated above.

Within each segment, however, the angular position of glance will be determined by a random variable, or a function of a random variable, to simulate the indeterminacy of the exact position of fixation points when visually searching.

The main problem to be solved here is the probability of visual detection of a moving target. Thus, it is required to determine whether or not the moving point source comes within the detection visual angle (or field of view) during the fixation period, and furthermore, to ascertain whether the point target is seen during the time it lies within the visual detection lobe limits while scanning. The probability of seeing, P_{see}, once the target is within the observable off-axis visual angle, depends on the effective contrast, C_{eff}, which is a direct function of the angular speed of the aircraft relative to the observer (if a fixed time of observation of target is assumed). A similar situation exists while the eye is scanning except that here, the major portion of the angular motion of a target on the retina is a result of the eyeball motion. The assumption is then made that no detection qualities of the eye are affected except those attributable to the rate effect of the image on the retina due to eye motion itself.

To elaborate on the method used to tackle this dynamic detection problem, reference must be made again to Figure 1. When the aircraft is located at point (X_{ij}, Y_{ij}), the distance from aircraft perpendicular to line of fixation of the i^{th} glimpse is d_{ij}. At the beginning of scan period, the separation distance is d'_{ij}. At any point in between, let the separation distance between aircraft and line of i^{th} glimpse be d. It is now required to compare whether the distance subtended by the off-axis visual angle, ξ , at range, R, is greater than separation distance, d. If so, there is a chance for detection. If not, no possibility for visual attention-getting is possible. Upon satisfying the above criteria for a chance of attention-getting, then the probability of seeing, P_{see}, which is a function of the effective contrast, C_{eff}, and the threshold contrast C_t must be included. However, the effective contrast, C_{eff}, depends strongly on the speed of the aircraft, and thus the detectability becomes a function of the dynamic changes occuring during the search. C_{eff} can also be written as $(\Delta I/I)_{eff}$ where I is the background intensity and $I = I_1$ is the increase or decrease of light intensity when the target obscures part of the background light.

A THEORETICAL MODEL FOR DETERMINING PROBABILITY OF VISUAL DETECTION

General Considerations

A statistical probability detection distribution as a function of range, R, and of time of exposure, τ, for each of the parameters is required. This can be determined by using a Monte Carlo approach.

To formulate the criterion of visual detection for a Monte Carlo approach, the problem must be stated in mathematical terms. This is done below:

The equation of the j^{th} flight path is

$$Y = (X - x_{oj}) \tan \phi_{oj} \tag{2}$$

Similarly the equation of the i^{th} glimpse becomes

$$Y = X \tan \theta_{ij} \tag{3}$$

Let the distance from point (X_{ij}, Y_{ij}) perpendicular to the line of fixation be d_{ij}.

Then $$d_{ij} = \frac{X_{ij} \tan \theta_{ij} - Y_{ij}}{(1 + \tan^2 \theta_{ij})^{1/2}} \tag{4}$$

Also, the perpendicular distance from point $(X_{ij} - vT_f \cos \phi_{oj}, Y_{ij} - vT_f \cos \phi_{oj})$ to the i^{th} line of fixation, d'_{ij} is

$$d'_{ij} = \frac{(X_{ij} - vT_f \cos \phi_{oj}) \tan\theta_{ij} - (Y_{ij} - vT_f \sin \phi_{oj})}{(1 + \tan^2\theta_{ij})^{1/2}} \tag{5}$$

In general, the instantaneous distance of aircraft from line of fixation is

$$d = \frac{X \tan\theta_{ij} - Y}{(1 + \tan^2\theta_{ij})^{1/2}} \tag{6}$$

Let there now be defined a quantity called the off-axis visual angle denoted by ξ . This is the maximum angular displacement of the target's position from the line of fixation within which visual perception can be considered greater than 50 percent. This off-axis angle as show: in Figure 2 is a function of several parameters and changes quite rapidly with range, R.

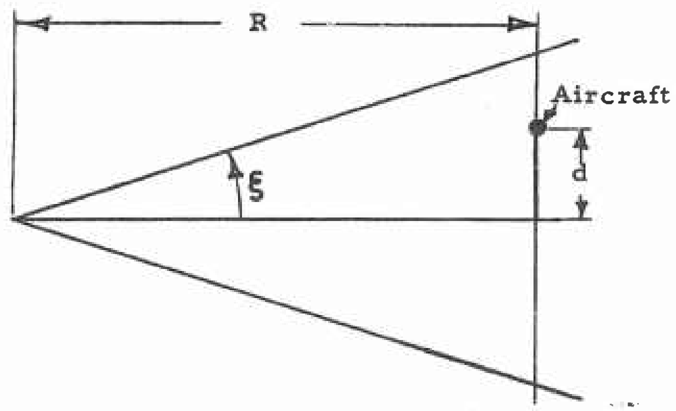

FIGURE 2. OFF-AXIS VISUAL ANGLE

It can be written that

$$R \tan\xi \geq d \tag{7}$$

is the condition that must be satisfied for a finite chance of detection to occur.

Also

$$R \triangleq (X^2 + Y^2)^{\frac{1}{2}} \tag{8}$$

To utilize the criterion given by equation (7), the functional variation of the off-axis visual angle, ξ, is required. A semi-mpirical formula has been developed which combines the bulk effects of

1 light transmissions in the atmosphere

2 reception of light by the retina

and uses these to define an off-axis perceptability region as a function of the physical parameters.

To the off-axis visual angle, ξ, are related the basic factors of presented area, range, intrinsic contrast and meteorological visibility (Reference 4); i. e.,

$$C_o e^{-3.44R/V} = 1.75\,\xi^{\frac{1}{2}} + 45.6\,R^2\xi/A_p \tag{9}$$

where

C_o	is the intrinsic target background contrast in percent
R	is the range in miles
V	is meteorological visibility in miles
ξ	is the off-axis visual angle in degrees
A_p	is the presented area of target in square feet

Solving the equation for ξ, we get

$$\xi = \left\{ \frac{\left[3.0625 + 182.4\,C_o(R^2/A_p)e^{-3.44R/V}\right]^{\frac{1}{2}} - 1.75}{91.2\,R^2/A_p} \right\}^2 \tag{10}$$

Equation (10) indicates that for a given set of physical parameters C_o, A_p, and V, the off-axis visual angle, ξ, is a function of range, R. Thus, the value of ξ at a given R, defines two points in the visual detection lobe. Repeating this for all ranges, R, we then define a visual detection lobe (Figure 3). All points within the detection lobe are assumed to have an equiprobable chance of attention-catching value equal to one (see Appendix).

Thus, even though acuity changes as one leaves the line of fixation, the off-axis visual angle, ξ, defines the angular limits where probability of perception is reasonably good. This is so everywhere within the limits of the detection lobe. This effectively takes into account the less sensitive parafoveal and peripheral regions of the retina as the target comes within its detection capability.

In order to know where the termination of the visual detection zone occurs, another relationship has been presented by the O. R. Evaluation Group of the Office of Naval Research (Reference 2).

Referring to the geometric situation (Figure 3), the maximum range of detection, R_m, can be expressed again in terms of the physical meteorological variables; i. e.,

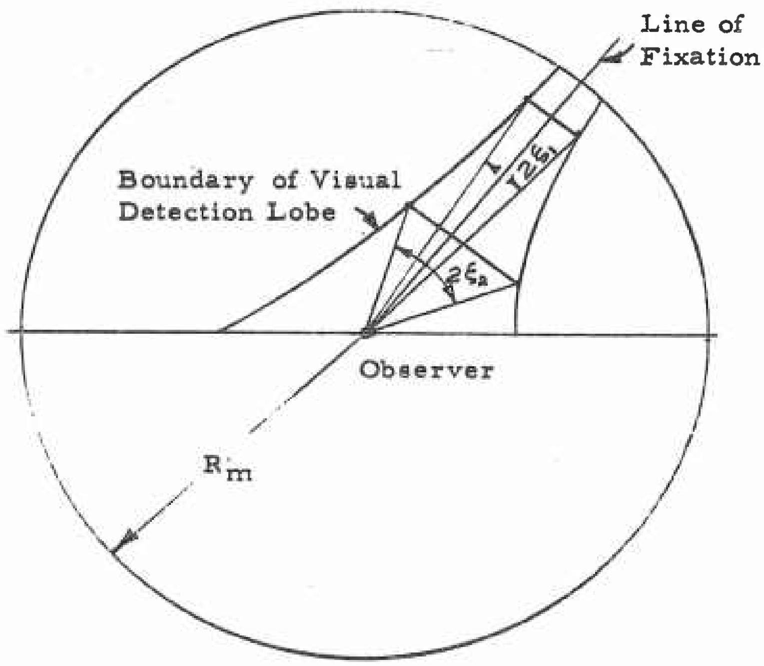

FIGURE 3. VISUAL DETECTION LOBE

$$R_m = (0.164)^2 A_p \left[C_c e^{-3.44R_m/V} - 1.57\right] \tag{11}$$

$$A_p = \frac{1}{R_m}\left[A_s d + A_b h + A_f (R_m^2 - d^2 - h^2)^{\frac{1}{2}}\right] \tag{12}$$

where

R_m	is the maximum detection range in miles
A_f	is the frontal projected area of aircraft in square feet
A_s	is the side projected area of aircraft in square feet
A_b	is the bottom projected area of aircraft in square feet
h	is the height of aircraft above ground level in feet

and where all others are as previously defined.

Thus, the distant boundary for visual detection varies with the meteorological conditions and with the visual properties of the target. The direction of glimpse at the time the aircraft crosses the boundary at R_m will be taken as the reference point for the definition of a random variable, Θ_{oj}, used in defining an initial direction of observer's search glimpse for the j^{th} flight path.

Using the formulas presented up to this point, the entire geometric situation is defined for a set position of the i^{th} glimpse. It can be determined whether the target satisfies the condition defined by equation (7) at any time. If it does satisfy the criterion, then the next step is to find the probability of seeing the target, P_{see}, given that it lies within the off-axis visual region given by equation (7).

Probability of Detection

Once a visual detection lobe has been defined by the following equation

$$d = R \tan \xi \tag{13}$$

it is next required to assign a probability of seeing, P_{see}, value for a target within the detection lobe. This step is necessary because many times in experimental situations, an observer will be looking right at the target and will fail to see it.

Lamar, et al. in Reference 3 conducted a comprehensive study on the detection of targets of different sizes, shapes, and contrast under laboratory conditions. In this paper they put forth their theory describing all their results on a quantum statistical footing. It, by and large, predicts experimental results very well.

Included in this theory is the derivation of the probability of seeing, P_{see}, as a function of the contrast, $\Delta I/I$. A brief summary of relevant theory and formulas leading up to this relationship is given in the following sections.

To explain the behavior of the human eye on a quantum statistical basis, the basic assumption used has been that in order for a target to become visible, a certain number of light quanta, n, must be absorbed in some manner and in some location by the cones.

To be more specific, it is assumed that the area of retina within which n or more quanta must be absorbed is further restricted to one or more sections of the narrow strip just inside the retinal image boundary. What this means is that the presence of a target will be recognized if one or more small segments of the perimeter each absorbs n or more light quanta.

To derive the probability of seeing, P_{see}, from the above hypothesis, the probability that at least n light quanta out of the M light quanta impinging on the retina will be absorbed in a given section of the boundary - i. e., P_n - must be found. The probability that less than n quanta will be absorbed in the given section is $1-P_n$. If there are K sections around the retinal image perimeter, then the probability that less than n quanta will be absorbed in each section is $(1-P_n)^K$. Therefore, the probability, P_{see}, that at least n quanta will be absorbed in at least one of the sections is

$$P_{see} = 1 - (1-P_n)^K \tag{14}$$

To find P_n, the probability of at least n successes out of M trials, use is made of Poisson's Law. If p denotes the probability of absorption of quanta per trial and M the total number of quanta impinging on the retina, then the expected number of successes, or absorptions of quanta by the retina, is

$$\epsilon = Mp \tag{15}$$

In terms of the expected number of successes, ϵ, the probability of exactly n successes in M trials is given by Poisson's Law as

$$P_n = (\epsilon^n e^{-\epsilon})/n! \tag{16}$$

The probability P_n of at least n successes out of M trials is given by

$$P_n = \sum_n^M (\epsilon^n e^{-\epsilon})/n! \tag{17}$$

Referring to equation (15), for finite values of ϵ, if p is small, M must be very large, so that equation (17) becomes

$$P_n = \sum_{n}^{\infty} (\epsilon^n e^{-\epsilon})/n! \tag{18}$$

Substituting equation (18) into equation (14),

$$P_{see} = 1 - \left[1 - \sum_{n}^{\infty} (\epsilon^n e^{-\epsilon})/n! \right]^K \tag{19}$$

When equation (19) is used to predict the experimental curve P_{see} vs $(\Delta I/I)_{eff}$, it is found that equation (19) describes the experimental curves best when

$n = 4$, the number of quanta absorbed for perception of target, and

$K \triangleq 10$, where K takes on the concept of the number of cones on the perimeter of retinal image.

Thus, equation (19) becomes

$$P_{see} = 1 - \left[1 - \sum_{4}^{\infty} (\epsilon^n e^{-\epsilon})/n! \right]^{10} \tag{20}$$

The expected average number of quanta absorbed in the retina, ϵ, is proportional to the effective contrast between target and background, $(\Delta I/I)_{eff}$, and therefore, P_{see} vs $(\Delta I/I)_{eff}$ can be obtained from equation (19).

The value chosen for K depends upon the perimeter of the retinal image but the variation in P_{see} is quite small for large variations in retinal perimeters. A value of K = 10 describes the average probability of seeing curve to within about 5 to 10 percent for values of retinal perimeter of interest.

Thus, it is seen that equation (19) is only a function of ϵ where

$$\epsilon = K_1 \, (\Delta I/I)_{eff} \tag{21}$$

and K_1 is a constant of proportionality.

Then,

$$P_{see} = 1 - \left\{ 1 - \sum_{4}^{\infty} \left[K_1 \, (\Delta I/I)_{eff} \right]^n e^{-K_1 (\Delta I/I)_{eff}} \Big/ n! \right\}^{10} \quad (22)$$

By substituting a point on the average probability of seeing curve, K_1, may be determined. Equation (22) then gives a functional description of P_{see} vs $(\Delta I/I)_{eff}$.

Because of individual differences in threshold levels of perception, the position of the P_{see} vs $(\Delta I/I)_{eff}$ curve shifts along the abscissa, the shape, however, remaining the same. To normalize the curve for general use, $(\Delta I/I)_{eff} - (\Delta I/I)_t$ is substituted for the abscissa instead of $(\Delta I/I)_{eff}$.

The contrast threshold, $(\Delta I/I)_t$, can then be obtained for an individual and substituted into equation (22).

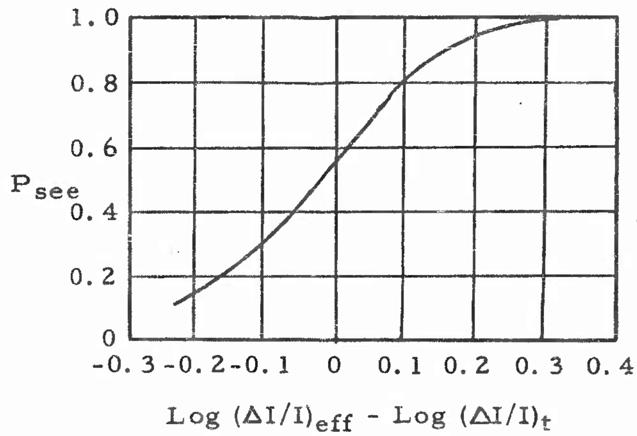

FIGURE 4. CHANCE OF PERCEPTION VS CONTRAST

The normalized curve

$$P_{see} \text{ vs } \log (\Delta I/I)_{eff} - \log (\Delta I/I)_t \quad (23)$$

shown in Figure 4 does not change appreciably with daylight to dusk intensities, perimeter of retinal image, and individual differences. It is a statistically averaged curve, good for all daylight detection conditions.

The value of contrast $(\Delta I/I)_{eff}$ to be used in equation (22) is the effective contrast at the retina of eye. Therefore, if the intrinsic contrast between target and background is C_o, then the value of $(\Delta I/I)_{eff}$ used in equation (22) is related to intrinsic contrast as follows (assuming target not moving in the sky):

$$C = (\Delta I/I)_{eff} = C_o\, e^{-3.912\, R/V} \qquad (24)$$

because of the attenuation due to the atmosphere.

Hence, $$(\Delta I/I)_{eff} - (\Delta I/I)_t = C_o\, e^{-3.912\, R/V} - (\Delta I/I)_t = C_1 \qquad (25)$$

where

C_1 is the resultant contrast to be used in determining probability of seeing,

and where all other symbols are as previously defined.

Equation (22) can be modified as follows:

$$P_{see} = 1 - \left[1 - \sum_{4}^{\infty} (K_1 C_1)^n\, e^{-K_1 C_1} \Big/ n!\right]^{10} \qquad (26)$$

What has been accomplished up to this point is the formulation of a criterion for the visual detection of point targets. If the target is within the detection lobe, the probability of seeing, P_{see}, must be determined; this depending upon the contrast, C, at the retina. On the other hand, if the target is outside the bounds of the visual detection lobe, the probability of seeing, P_{see}, is taken to be uniformly zero.

The next major step required is to incorporate quantitatively the visual detection characteristics of a moving target. This involves a rate of movement in the retinal image of the target and hence the integrating capacity of the eye must be considered. This dynamic aspect to the target detection problem will now be examined.

Detection of Moving Targets

The extension of the visual detection analysis to include the dynamic effects of high-speed aircraft was made possible upon finding, and then examining, a theoretical treatment of contrast thresholds of moving-point sources.

In Reference 4 it was found that the variable which is effectively altered under dynamic conditions was M, the total number of light quanta absorbed by the retina. The quantity M, as will be shown later, is directly proportional to the contrast, C, at the eye.

It was shown in the previous section of this report that the probability of seeing, P_{see}, given that target is within the perceptibility region, was a universal function of the resultant contrast, C_1. Keeping in mind that the total energy from a target impinging on the retina, M, is proportional to the contrast, C, the theory applying to stationary targets may be extended to that which included the dynamic variables influencing the visual detection of moving targets. This can be done by inserting the changes due to motion into the resultant contrast, C_1, which, in turn, is the abscissa in the universal curve P_{see} vs C_1, in Figure 4. In this way the probability of seeing, P_{see}, value is modified to include the motion of the target.

Proceeding with the above-mentioned approach of including the dynamic effects of detection and the quantitative relationships for the energy effectively available for absorption when the retinal image has a velocity on the retina may be determined. A general statement of the quantitative relationship between energy (number of light quanta) required for perception of light and the angular velocity of the target can be stated as follows:

For a fixed flash time of target the number of light quanta from the target impinging on the retina

$$M \sim \text{Constant when } (d\vartheta/dt)\,\tau_e \leqslant \delta \qquad \text{(a)}$$

and (27)

$$M \sim (d\vartheta/dt)^{(n-1)/n} \text{ when } (d\vartheta/dt)\,\tau_e \gg \delta \qquad \text{(b)}$$

where

τ_e is the time constant of the eye within which time the integration probability of the retina approaches one (in seconds)

δ is the distance on the retina within which the integration probability of the retina approaches one (in minutes).

n — is the number of light quanta above the background intensity required for the perception of light if light quanta are absorbed within τ_e and δ of each other

$\frac{d\vartheta}{dt}$ — is the retinal angular velocity of target's image in minutes/seconds.

Equation (27b) applies if the condition $d\vartheta/dt \cdot \tau_e \gg \delta$ holds, and therefore, the relationship of M to $d\vartheta/dt$ is actually a description of the asymptote to the exact curve when it is near $d\vartheta/dt = \delta/\tau_e$. Thus, the asymptote relating M to $d\vartheta/dt$ originates at $d\vartheta/dt = \delta/\tau_e$ and rises with a constant slope $(n-1)/n$ if plotted in log-log coordinates.

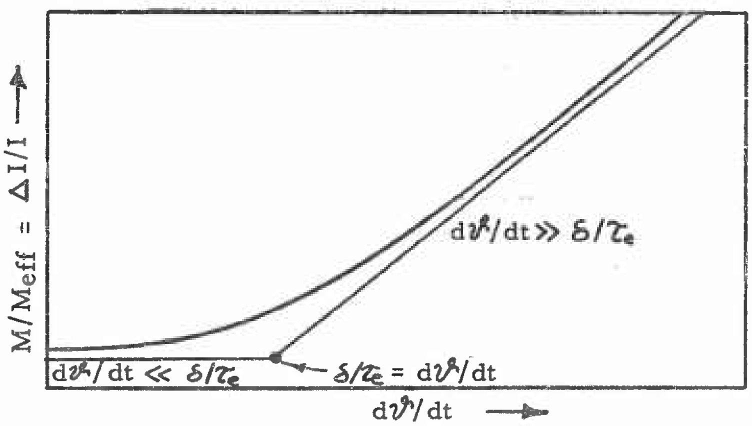

FIGURE 5. ADDITIONAL LIGHT ENERGY REQUIRED WITH ANGULAR VELOCITY

By utilizing the equations of the asymptotes rather than the exact curve, the proportionality constants can be inserted into equation (27) as follows:

$$M = M_{eff}\left(\frac{d\vartheta}{dt}\right)^{\frac{n-1}{n}} \Bigg/ \left(\frac{\delta}{\tau_e}\right)^{\frac{n-1}{n}} \quad \text{when} \quad \frac{d\vartheta}{dt} \gg \frac{\delta}{\tau_e}$$

$$M = M_{eff} \quad \text{when} \quad \frac{d\vartheta}{dt} \ll \frac{\delta}{\tau_e} \tag{28}$$

where

M_{eff} is the number of light quanta impinging on the retina that fall within the integration capability of the retina when source of quanta is moving,

and where all others are as previously defined.

Using the relationship given in equation (15), the expected average number of absorptions of quanta (or successes), ϵ', can be related to M, the total number of quanta impinging on retina, and p, the probability of absorption of a given quantum.

$$\epsilon' = Mp \tag{29}$$

$$\epsilon_{eff} = pM_{eff} \tag{30}$$

where

ϵ' is the expected average number of absorptions of light quanta from a moving target falling on the retina, and

ϵ_{eff} is the effective expected average number of absorptions of light quanta from a moving target which fall within the integrating capacity of the eye.

Using equations (29) and (30), we can convert equation (28) to the following:

$$\epsilon' = \epsilon_{eff}\,(d\nu/dt)^{(n-1)/n} \Big/ (\delta/\tau_e)^{(n-1)/n} \quad \text{when } (d\nu/dt)\,\tau_e > \delta$$

$$(31)$$

$$\epsilon' = \epsilon_{eff} \quad \text{when } (d\nu/dt)\,\tau_e < \delta$$

Equation (31) has been formulated for a fixed time of observation, but in this problem the time that the aircraft remains within the detection lobe changes. In fact, under the high-speed conditions of target motion, the time that the aircraft remains within the detection lobe can be represented as different durations of flash intensities.

An equation by Blondel and Rey mentioned in Reference 5 gives a relationship between a flash-type signal and a steady signal.

Since a steady signal is one whose duration is large compared to τ_e and the experimental method which will be utilized in assigning a value for n pre-supposes an equivalently steady signal, the equation can be incorporated into the proposed model to give dependence on flash durations of target.

This equation can be written as follows:

$$(\Delta I')/(\Delta I) = (K\tau_f)/(k + \tau_f) \qquad K = 1, \quad k = 0.21 \tag{32}$$

where

$\Delta I'$ is the effective differential intensity of a flashing light

ΔI is the effective differential intensity of a steady light

τ_f is the duration on-phase of the flashing signal or the duration that the aircraft remains within the perceptibility region.

Also, from equation (22) it is observed that ϵ, the expected average number of absorptions, is proportional to retinal illumination $\Delta E/E$ and hence to contrast $\Delta I/I$ at the retina:

$$\text{i.e.,} \qquad \epsilon = K_o(\Delta E/E) = K_1(\Delta I/I) \tag{33}$$

where K_o and K_1 are constants.

Keeping the above relationships in mind and utilizing the results stated in equation (32),

$$(\epsilon')/(\epsilon) = (\Delta I'/I')/(\Delta I/I) = (\tau_f)/(0.21 + \tau_f) \qquad (\text{Since } I' = I) \tag{34}$$

Utilizing equations (31) and (33), ϵ_{eff}, the effective expected average number of absorptions, may be related to ϵ, the expected average number of absorptions, when both image velocities and flash durations are taken into account.

$$\frac{\epsilon_{eff}}{\epsilon} = \frac{\epsilon_{eff}}{\epsilon'} \cdot \frac{\epsilon'}{\epsilon} = \left(\frac{\delta/\tau}{d\nu/dt}\right)^{\frac{n-1}{n}} \cdot \frac{\tau_f}{0.21 + \tau_f}$$

so that

$$\left.\begin{aligned} \frac{\epsilon_{eff}}{\epsilon} &= \left(\frac{\delta/t}{d\vartheta/dt}\right)^{\frac{n-1}{n}} \cdot \frac{\tau_f}{0.21+\tau_f} && \text{when } \frac{d\vartheta}{dt}\cdot\tau_e \gg \delta \\ &= \frac{\tau_f}{0.21+\tau_f} && \text{when } \frac{d\vartheta}{dt}\cdot\tau_e < \delta \end{aligned}\right\} \tau_f > \tau_e \qquad (35)$$

Utilizing equation (33), equation (35) can be converted into the following form:

$$\begin{aligned} \frac{C_{eff}}{C} &= \left(\frac{\delta/\tau_e}{d\vartheta/dt}\right)^{\frac{n-1}{n}} \cdot \frac{\tau_f}{0.21+\tau_f} && \text{when } \frac{d\vartheta}{dt}\cdot\tau_e \gg \delta \\ \frac{C_{eff}}{C} &= \frac{\tau_f}{0.21+\tau_f} && \text{when } \frac{d\vartheta}{dt}\cdot\tau_e \leq \delta \end{aligned} \qquad (36)$$

The contrast, C, can be further related to the intrinsic contrast between target and background. This is done by introducing an exponential attenuation factor because of atmosphere scattering and refraction as light quanta travel towards the retina.

$$C = C_o e^{-3.44R/V} \qquad (37)$$

Having related the effective contrast, C_{eff}, involving retinal velocity and flash time of target observation to the intrinsic contrast, C_o, a physical characteristic of an aircraft under typical sky background, it remains to obtain realistic values for n, $d\vartheta/dt$, τ_e and δ under daylight adapted conditions of the eye. Each of the quantities will be considered separately and a value will be assigned where possible.

Value for n

As mentioned earlier in this volume in connection with the derivation of the probability of seeing, P_{see}, the number of light quanta, n, that must be absorbed within τ_e and δ under daylight adapted conditions was found to be 4 for the perception of a differential light intensity (Reference 5).

Therefore, n = 4 in the relationship established in equation (37).

Value for τ_e

In a paper by M. A. Bouman (Reference 6), it was found that the time constant of integration for the eye can be taken as τ_e = 0.05 seconds with a high degree of accuracy. This value for τ_e is independent of the wavelength and even the location on the retina upon which light impinges.

Value for δ

The value for δ, the effective length on retina within which 4 light quanta must be absorbed in order to perceive an increment of light, depends on the wavelength of light and on retinal position upon which light falls (see Reference 2 for details).

However, under daylight-to-dusk light intensities, Lamar, et al. in Reference 3 indicate that the effective value for δ is approximately the cone diameter when target detection is studied for a daylight distribution of wavelengths; i.e., δ = 0.55 minutes of arc.

However, it is quite conceivable that δ can vary substantially from this value, depending on the wavelength or wavelengths reflected by target.

The variation of δ with retinal position is assumed negligible because the main detecting area of the retina is the foveal region, and there, the change is negligible.

To derive an approximation for the time that the aircraft remains in perception region,

let d_{ij_1} be the distance of aircraft position perpendicular to line of fixation when it intercepts the lobe boundary

d'_{ij_1} be the distance of aircraft position perpendicular to line of fixation when it leaves the bounds of the detection lobe

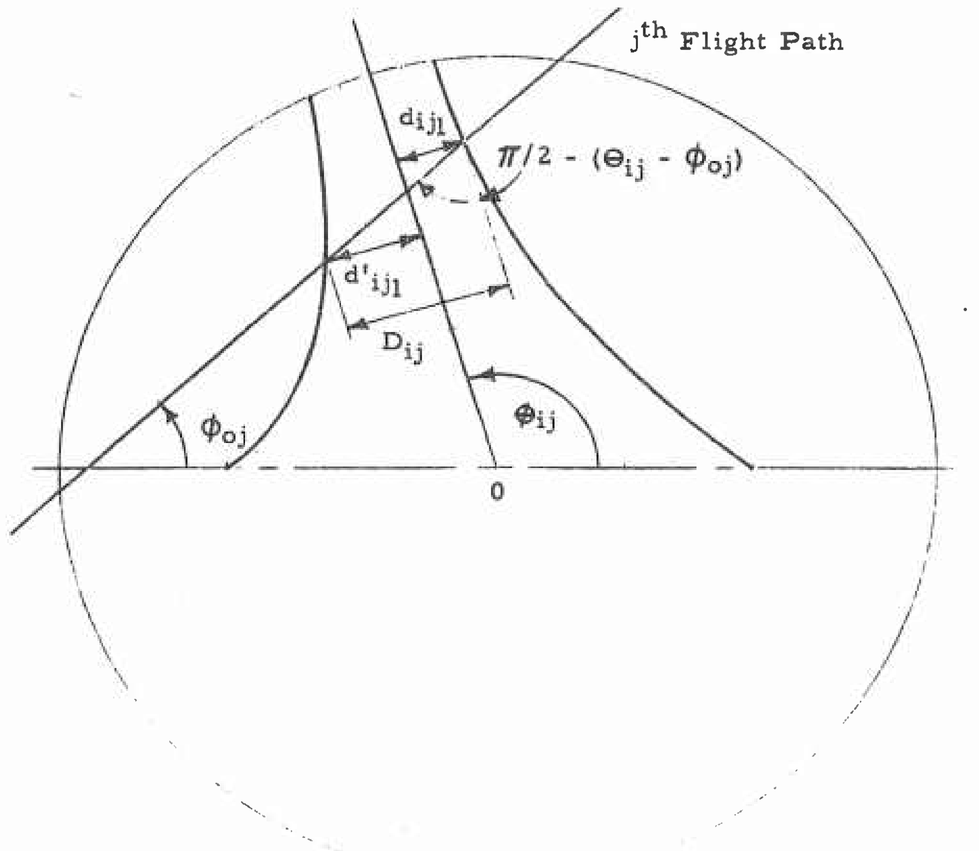

FIGURE 6. GEOMETRICAL RELATIONSHIPS REQUIRED TO DETERMINE THE AIRCRAFT'S TIME OF EXPOSURE DURING A GLIMPSE

D_{ij} be the total distance aircraft traverses while within detection lobe

R_1 be the range upon entering the detection lobe

R_1' be the range upon leaving the detection lobe,

and let all others be as previously defined.

$$D_{ij} = (d_{ij_1} + d'_{ij_1}) \cos \left[\pi/2 - (\theta_{ij} - \phi_{oj})\right] \tag{38}$$

Also,
$$d_{ij_1} = R_1 \tan \xi, \quad d'_{ij_1} = R_1' \tan \xi' \tag{39}$$

$$D_{ij} = \left(R_1 \tan \xi + R_1' \tan \xi'\right) \sin (\theta_{ij} - \phi_{ij}) \tag{40}$$

$$\xi_1 = \left\{\frac{\left[3.0625 + 182.4\, C_o (R_1{}^2/A_p)\, e^{-3.44 R_1/V}\right]^{\frac{1}{2}} - 1.75}{91.2\, (R_1{}^2/A_p)}\right\}^2 \tag{41}$$

$$\xi_1' = \left\{\frac{\left[3.0625 + 182.4\, C_o (R_1'{}^2/A_p)\, e^{-3.44 R_1'/V}\right]^{\frac{1}{2}} - 1.75}{91.2\, (R_1'{}^2/A_p)}\right\}^2 \tag{42}$$

Therefore, the time during which the target is effectively within the perceptibility region for i^{th} glance and j^{th} flight path is

$$\tau_{fij} = D_{ij}/v = (1/v)\,(R_1 \tan \xi_1 + R_1' \tan \xi_1') \sin (\theta_{ij} - \phi_{oj}) \tag{43}$$

Value for $d\vartheta/dt$

To apply equation (36), it is necessary to obtain $d\vartheta/dt$ as a function of aircraft velocity and aircraft position co-ordinates.

For the aircraft's instantaneous position (X, Y) as shown in Figure 7,

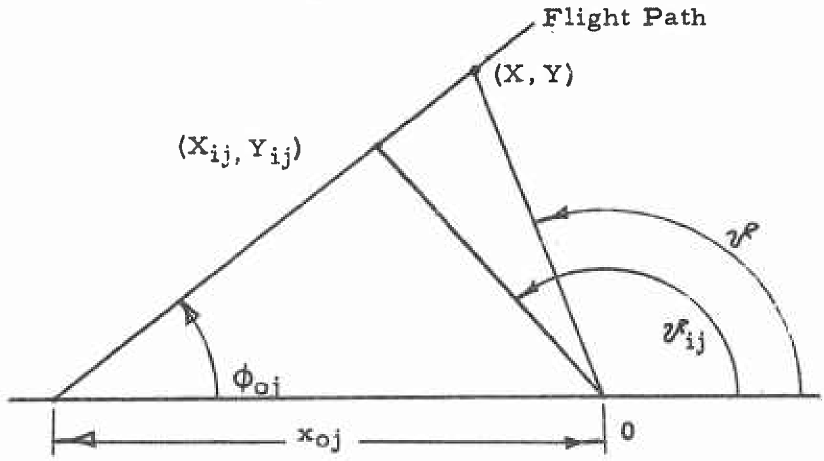

FIGURE 7. ANGULAR VELOCITY RELATIVE TO OBSERVER

$$\vartheta = 57.3 \times 60 \tan^{-1} Y/X \tag{44}$$

$$\frac{d\vartheta}{dt} = 57.3 \times 60 \frac{Xv_y - Yv_x}{X^2 + Y^2} \tag{45}$$

where

v_x and v_y are velocities along the X and Y axis

$d\vartheta/dt$ is the rate of movement of retinal image in minutes/second.

Armed with discrete values of τ_e, δ, n, and function value of τ_f and $d\vartheta/dt$ together with the foregoing comments as to the variations of the above parameters, equation (36) becomes:

$$C_{eff} = C_o e^{-3.44R_1/V \left(\frac{\frac{11}{57.3 \times 60} R_1^2}{v_yX - v_xY} \right)^{3/4}} \frac{(R_1 \tan \xi_1 + R_1' \tan \xi_1')\sin(\Theta - \phi)}{(R_1 \tan \xi_1 + R_1' \tan \xi_1')\sin(\Theta - \phi) + 0.21v} \tag{46a}$$

when $d\vartheta/dt > \delta/\tau_e$ = 11 minutes/second.

$$C_{eff} = C_o e^{-3.44 R_1/V} \frac{(R_1 \tan \xi_1 + R_1' \tan \xi_1') \sin (\Theta - \Phi)}{(R_1 \tan \xi_1 + R_1' \tan \xi_1') \cos (\Theta - \Phi) + 0.21 v} \quad (46\ b)$$

when $d\vartheta/dt \leqslant \delta/\tau_e$ = 11 minutes/second.

Separation of Visual Detection Area into Regions $d\vartheta/dt > \delta/\tau_e$ & $d\vartheta/dt < \delta/\tau_e$

To find the geometric regions within which $\tau_e\, d\vartheta/dt \geqslant \delta$, equation (45) is manipulated by substituting $v \sin \Phi_{oj}$ for v_y and $v \cos \Phi_{oj}$ for v_x:

$$d\vartheta/dt = 57.3 \times 60\ v\ (X \sin \Phi_{oj} - Y \cos \Phi_{oj})/(X^2 + Y^2) \quad (47)$$

Then, $57.3 \times 60\ v\, \tau_e\ (X \sin \Phi_{oj} - Y \cos \Phi_{oj})/(X^2 + Y^2) \geqslant \delta$

is the region which must be satisfied, where δ is in minutes.

$$57.3 \times 60\ v\left(\frac{\tau_e}{\delta}\right)(X \sin \Phi_{oj} - Y \cos \Phi_{oj}) \geqslant X^2 + Y^2$$

$$X^2 - 3438\ v\left(\frac{\tau_e}{\delta}\right) X \sin \Phi_{oj} + \left[1719\ v\left(\frac{\tau_e}{\delta}\right) \sin \Phi_{oj}\right]^2 + Y^2 +$$

$$3438\ v\left(\frac{\tau_e}{\delta}\right) Y \cos \Phi_{oj} + \left[1719\ v\left(\frac{\tau_e}{\delta}\right) \cos \Phi_{oj}\right]^2 \leqslant \left(1719\ v \frac{\tau_e}{\delta}\right)^2$$

$$\left[X - 1719\ v\left(\frac{\tau_e}{\delta}\right) \sin \Phi_{oj}\right]^2 + \left[Y + 1719\ v\left(\frac{\tau_e}{\delta}\right) \cos \Phi_{oj}\right]^2 \leqslant \left(1719\ v \frac{\tau_e}{\delta}\right)^2 \quad (48)$$

The locus described by equation (48) is a circle with center at $\left[1719\ v\left(\frac{\tau_e}{\delta}\right) \sin \Phi_{oj},\ -1719\ v\left(\frac{\tau_e}{\delta}\right) \cos \Phi_{oj}\right]$ and radius of length $1719 \cdot v\, \tau_e/\delta$.

This locus gives the boundaries separating the area into regions where $d\vartheta/dt > \delta/\tau_e$ and $d\vartheta/dt < \delta/\tau_e$, different effective contrasts, C_{eff}, applying in each of these regions [see equation (46)].

Assuming a typical value of the linear velocity of the aircraft (i.e., v = 1000 feet/second or 1/5 mile/second), an order of magnitude for the radius of the circle can be found.

Radius of circle = $1719\ v\, \tau_e/\delta$ = 34.3 miles

Then,

$$\text{Center of circle} = 34.3 \sin \Phi_{oj},\ - 34.3 \cos \Phi_{oj} \text{ miles}$$

These regions are drawn graphically for the j^{th} flight path and i^{th} glimpse in Figure 8.

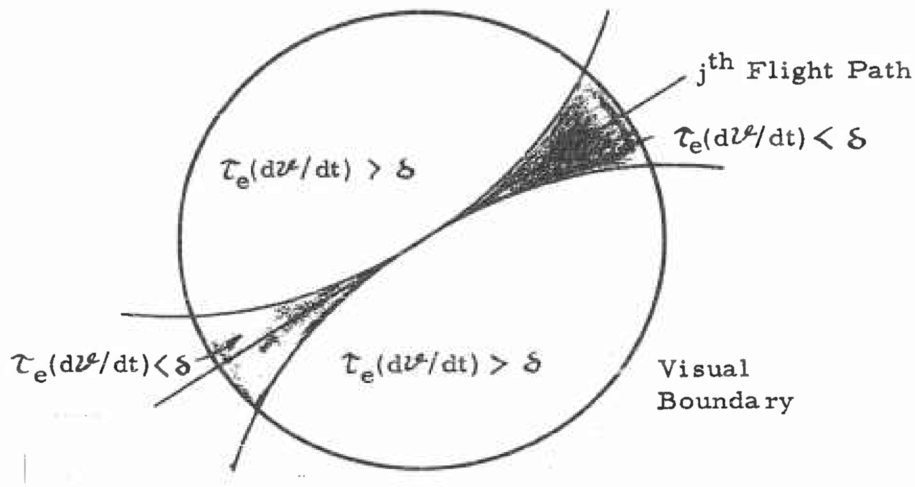

FIGURE 8. REGIONS WHERE ANGULAR VELOCITIES ARE GREATER THAN AND LESS THAN δ/τ_e

It is seen that most of the area in which visual detection is possible involves the condition $(d\vartheta/dt)\tau_e > \delta$, and therefore. equation (46a) can be used uniformly to obtain C_{eff}.

All the concepts and formulas considered have been directed towards describing the visual detection ability of the fixated eye looking for a moving target. The same concepts can also be applied to the scanning search mode of the eyes. It is assumed that the retinal angular velocity of the target is due entirely to the relative angular velocity of the observer's eyes and that no other acuity changes occur in the retinal detection ability.

Then in equation (36) $d\vartheta/dt$ can be replaced by $d\Theta/dt$ in degrees, the angular rate of observer's scan; i.e.,

$$C_{eff} = C_o e^{-3.44\, R/V} \,(11/3438\ d\Theta/dt)^{3/4}\ (\tau_p/0.21 + \tau_p) \text{ when } (d\vartheta/dt)\tau_e > \delta. \qquad (49)$$

Here $d\Theta/dt$ can be chosen according to any scan angular velocity function; i.e., $d\Theta/dt$ = constant is the time of observation of target (time within detection lobe limits during scanning).

$$T_m = \frac{2\xi}{d\Theta/dt} = \frac{2}{d\Theta/dt} \left\{ \frac{\left[3.0625 + 182.4\, C_o (R^2/A_p)\right]^{\frac{1}{2}} - 1.75}{91.2\, R^2/A_p} \right\}^2 \tag{50}$$

Description of Glimpse and Scan Behavior by Observer

From the following two modes of possible detection,

1) the fixated glimpse mode in which the maximum acuity of the eye is utilized (the achievement of maximum acuity is coupled with a relatively small solid angle of coverage per unit time), and

2) the scanning mode in which acuity is reduced (but on the other hand, the solid angle swept can be extremely large) and which is required to define a scanning-fixate pattern for target search,

a pattern of behavior of a human observer under alerted type conditions will be postulated. The main properties that will be included in the scanning-glimpse pattern will be as follows:

1) The necessity of including a randomness in the selection of the line of visual fixation of the observer.

2) The characteristic of most human observers to glimpse at an angle displaced from original glimpse by a most probable angular displacement.

A function which is realistic and yet fairly simple can be defined to help position successive fixated lines of vision.

This can be written on an empirical basis as follows:

$$\Theta_{i+1,j} - \Theta_{ij} = (\pi/2)\sin(2\,\chi_{ij} - 1)(\pi/2) \text{ when } 0 \leq \Theta_{i+1,j} \leq \pi \tag{51}$$

where

$\Theta_{i+1,j}$ is the angular position of glimpse relative to positive X axis for i + 1st glimpse, j^{th} flight path

Θ_{ij} is angular position of glimpse for i^{th} glimpse j^{th} flight path

χ_{ij} is a uniformly random variable defined between 0 and 1.

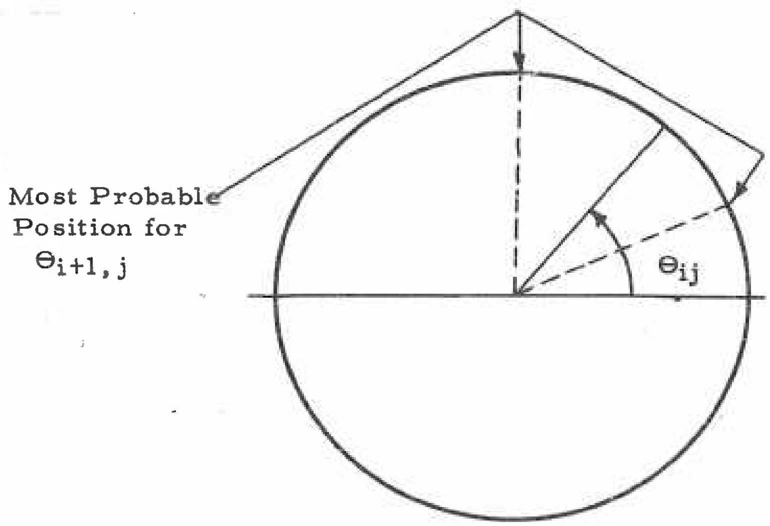

FIGURE 9. POSITION OF OBSERVER'S MOST LIKELY GLANCE

By choosing a random number between 0 and 1 for χ_{ij}, we get a most likely position for the successive glimpse according to a sinusoidal function (Figure 9).

To keep this dynamic analysis simple, it is further assumed that the rate of angular rotation when scanning towards a new fixation is constant; i. e.,

$$d\Theta/dt = c \tag{52}$$

Hence, the time that the target remains within the perception region is constant as the target image moves across the retina.

$$\text{i. e., } C_{eff} = (11/c)^{3/4} \cdot \tau_f/(0.21 + \tau_f) \tag{53}$$

A quick calculation indicates that any value of $d\Theta/dt$ between 11 minutes/second up to a maximum of 15 degrees/second is within detection possibility when scanning if the intrinsic contrast between target and background is of the order of 30 percent.

The value of T_m, time of scanning period, thus changes for each glimpse change depending upon angular distance traversed by the observer if equation (41) is obeyed.

Any alternative assumption of observer's behavior can be included if more definite information is obtained.

Method To Be Used in Constructing P_d vs R and P_d vs τ Curves

By using a Monte Carlo approach on the computer for random flight paths, the number of detections, N, between each range, R, and R $+\Delta$R can be accumulated and stored. From this information, a histogram of the number of detections, N, may be made between R and R $+ \Delta$R for each range, R.

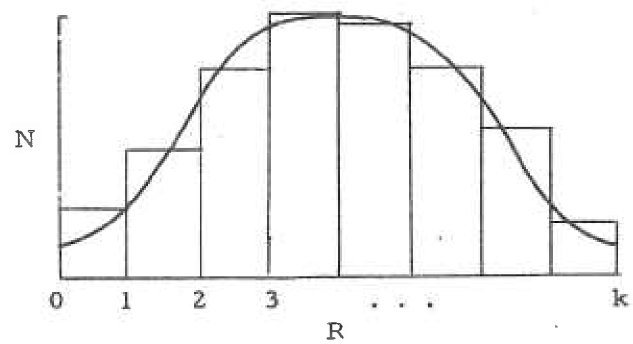

FIGURE 10. HISTOGRAM OF FREQUENCY OF DETECTION AS A FUNCTION OF RANGE

To obtain a probability of visual detection curve as a function of range, R, each N must be weighted with a probability of seeing quantity, P_{see}, because each possible detection has a definite probability of detection. For the k^{th} interval of histogram,

$$P_{d_k} = \sum_{\ell=1}^{N_k} P_{see\ k\ell} \Big/ N_k \qquad (54)$$

where

P_{d_k} is the probability of detection for the k^{th} interval in the histogram

$P_{see\ k\ell}$ is the probability of seeing for the ℓ^{th} point in the k^{th} section of the histogram

N_k is the number of points in the k^{th} section of the histogram

The exact subdivision required to obtain a well defined probability of detection curve P_d vs R is not known. However, sufficient resolution should be obtained if intervals in R are about 1,000 yards..

To obtain probability of detection, P_d, vs the time of exposure, τ, τ must be related to R.

Defining $\tau = 0$ when $R = R_m$ (see Figure 11),

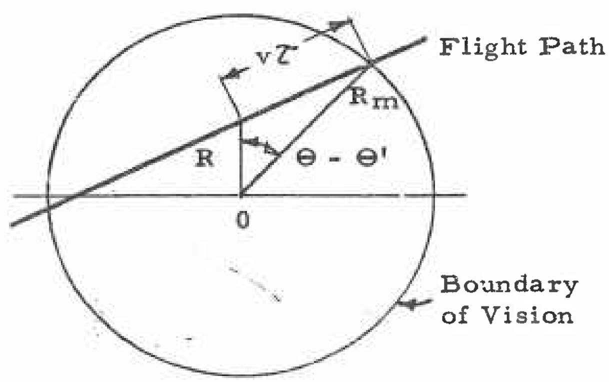

FIGURE 11. CONSTRUCTION TO DETERMINE TIME OF EXPOSURE IN SEARCH AREA

from the triangle AOB, $(v\tau)^2 = R^2 + R_m{}^2 - 2RR_m\cos(\Theta - \Theta')$. (55)

If detection occurs at range, R, then time of exposure, τ, can be derived from equation (43).

$$\tau = (1/v)\left[R^2 + R_m{}^2 - 2RR_m\cos(\Theta - \Theta')\right]^{\frac{1}{2}} \qquad (56)$$

where

τ is the time of exposure of the target from the point it enters the visual detection field to point of detection which is equivalent to the time elapsed since aircraft entered detection field to the time of detection

Θ is the aircraft's angular position relative to the observer in degrees.

Substituting for Θ and Θ'

$$\tau^2 = (1/v^2)\left[R^2 + R_m{}^2 - 2RR_m\cos(\tan^{-1} Y/X - \tan^{-1} Y_m/X_m)\right] \qquad (57)$$

$$\tau = (1/v)\left[R^2 + R_m{}^2 - 2RR_m\cos(\tan^{-1} Y/X - \tan^{-1} Y_m/X_m)\right]^{\frac{1}{2}} \qquad (58)$$

where

$$R_m{}^2 = Y_m{}^2 + X_m{}^2.$$

Thus, every time a finite probability of seeing, P_{see}, occurs at range, R, then by using equation (19) a $P_{see\,k\ell}$ corresponding to each τ may be determined.

The probability of detection, P_{dk}, can be determined for the interval in histogram between τ and $\tau + \Delta\tau$ by the following:

$$P_{dk} = \sum_{\ell=1}^{N_k} P_{see_{k\ell}} \Big/ N_k \qquad (59)$$

From the outlines of the above model, the probability of detection, P_d, vs range, R, and also the probability of detection, P_d, vs time of exposure, τ, as a function of all the parameters involved may be obtained.

APPLICATION OF THEORY INTO THE PROBLEM OF SURVIVABILITY OF MANNED AIRCRAFT

One of the probability factors in the overall probability of kill is the probability of detection, given line of sight, $P(D/L_s)$ (see Volume IV). In this case line of sight to the target has been assumed right from the start.

The results that can be obtained from the histogram, i.e., P_d vs R, are thus the ones representing $P(D/L_s)$ for the visual detection of aircraft. They can be utilized in determining the probability of kill, P(K), of an aircraft when using vision for initial detection of the target.

BIBLIOGRAPHY

Reference

1 Wokoun, William, Detection of Random Low-Altitude Jet Aircraft by Ground Observers, Technical Memorandum 7-60, OCO Guided Missile System Branch, U.S. Army Ordnance Human Engineering Laboratories, Aberdeen Proving Ground, Maryland, June 1960.

2 Operation Evaluation Group, Visual Detection in Air Interception: A Comparison of Theory With Trial Results, Study No. 470 (Revised), Office of the Chief of Naval Operations, Washington, D.C., April 1952.

3 Lamar, Edward S., et al., "Size, Shape, and Contrast in Detection of Targets by Daylight Vision. II. Frequency of Seeing and the Quantum Theory of Cone Vision", Journal of the Optical Society of America, Volume 38, Number 9, September 1948, pp. 741 to 755.

4 Bouman, M. A., and van den Brink, G., "Absolute Thresholds for Moving Point Sources", Journal of the Optical Society of America, Volume 43, Number 10, October 1953, pp. 895 to 898.

5 Gerathewohl, Siegfried J., "Conspicuity of Steady and Flashing Light Signals: Variations of Contrast", Journal of the Optical Society of America, Volume 43, Number 7, July 1953, pp. 567 to 571.

6 Bouman, M. A., and van den Brink, G., "On the Integrate Capacity in Time and Space of the Human Peripheral Retina", Journal of the Optical Society of America, Volume 42, Number 9, September 1952, pp. 617 to 620.

Other Articles Consulted are Listed Below

Hurvick, Leo M., and Jameson, Dorothea, "Spectral Sensitivity of the Fovea. I. Neutical Adaptation", Journal of the Optical Society of America, Volume 43, Number 6, June 1953, pp. 485 to 494.

Hurvich, Leo M., and Jameson, Dorothea, "Spectral Sensitivity of the Fovea. II. Dependence on Chromatic Adaptation", Journal of the Optical Society of America, Volume 43, Number 7, July 1953, pp. 552 to 559.

Rose, Alberts, "The Sensitivity Performance of the Human Eye on Absolute Scale", Journal of the Optical Society of America, Volume 38, Number 2, February 1948, pp. 196 to 208.

Kincaid, William, "Neural Formulation of the Effects of Target Size and Shape upon Visual Detection", Journal of the Optical Society of America, Volume 50, Number 2, February 1960, pp. 143 to 148.

Lichtenstein, M., and Boucher R., "Minimum Detectable Dark Interval between Trains of Perceptually Fused Flashes", Journal of the Optical Society of America, Volume 50, Number 5, May 1960, pp. 461 to 466.

McCalgin, Franklin, H., "Movement Thresholds in Peripheral Vision", Journal of the Optical Society of America, Volume 50, Number 8, August 1960, pp. 774 to 779.

Bouman, M. A., and van der Velden, H. A., "The Two-Quanta Hypothesis as a General Explanation for the Behavior of Threshold Values and Visual Acuity for the Several Receptors of the Human Eye", Journal of the Optical Society of America, Volume 38, Number 7, July 1948, pp. 570 to 581.

Simonson, Ernst, and Brozek, Josef, "The Effect of Spectral Quality of Light on Visual Performance and Fatigue", Journal of the Optical Society of America, Volume 38, Number 10, October 1948, pp. 830 to 840.

Simonson, Ernst, and Brozek, Josef, "Effects of Illumination Level on Visual Performance and Fatigue", Journal of the Optical Society of America, Volume 38, Number 4, April 1948, pp. 384 to 397.

Tanner, Jr., Wilson P., and Swets, John, A., "A Decision Making Theory of Visual Detection", Psychological Review, Volume 61, Number 6, 1954, pp. 401 to 409.

Bouman, M.A., "Mechanisms in Peripheral Dark Adaptation" Journal of the Optical Society of America, Volume 42, Number 12, December 1952, pp. 941 to 950.

Davy, Early, "The Intensity-Time Relation for Multiple Flashes of Light in the Peripheral Retina", Journal of the Optical Society of America, Volume 42, Number 12, December 1952, pp. 937 to 940.

Gibson, James J., "The Relative Accuracy of Visual Perception of Motion During Fixation and Pursuit, The American Journal of Psychology, Volume 70, 1957, pp. 64 to 68.

Gibson, James J., "The Visual Perception of Objective Motion and Subjective Movement", Psychological Review, Volume 61, Number 5, 1954, pp. 304 to 314.

Attneave, Fred, "Some Informational Aspects of Visual Perception", Psychological Review, Volume 61, Number 3, 1954, pp. 183 to 193.

Cohen, Walter, "Spatial and Textural Characteristics of the Ganzfeld", American Journal of Psychology, Volume 70, 1957, pp. 64 to 68.

Mull, Helen K., et al., "The Effect of Two Brightness Factors Upon the Rate of Fluctuation of Reversible Perspectives". (Source undetermined)

Krendel, Ezra, S., and Wodinsky, Jerome, "Search in an Unstructured Visual Field", Journal of the Optical Society of America, Volume 50, Number 6, June 1960, pp. 562 to 568.

Burnham, R. W., and Newhall, S. M., "Some Observations and Theory on the Variation of Visual Acuity with the Orientation of the Test Object", Journal of the Optical Society of America, Volume 43, Number 10, October 1953, pp. 902 to 905.

Nachman, Marvin, "The Influence of Size and Shape on the Discrimination of Visual Intensity", American Journal of Psychology, Volume 70, 1957, pp. 211 to 218.

Smith, Olin W., "A new Exploration of the Velocity Transporation", American Journal of Psychology, Volume 70, 1957, pp. 102 to 105.

Casperson, Roland Carl, "The Visual Discrimination of Geometric Forms", Journal of Experimental Psychology, Volume 40, October 1955, pp. 668 to 681.

Bouman, M. A., and van der Velden, H. A., "The Two-Quanta Explanation of the Dependence of the Threshold Values and Visual Acuity on the Visual Angle and the Time of Observation", Journal of the Optical Society of America, Volume 37, Number 11, November 1947, pp. 908 to 919.

Brown, John L., et al. "Effect of Duration of Light Adaptation on Time Required for Detection of a Target on a Simulated PPI Scope", Journal of the Optical Society of America, Volume 43, Number 12, December 1953, pp. 1147 to 1152.

Enoch, Joy M., "Response of a Model Retinal Receptor As a Function of Wavelength", Journal of the Optical Society of America, Volume 50, Number 4, April 1960, pp. 315 to 320.

Ogle, Kenneth, N., "Blurring of the Retinal Image and Contrast Thresholds in the Fovea", Journal of the Optical Society of America, Volume 50, Number 4, April 1960, pp. 307 to 314.

Hanson, John A., et al., "Studies on Dark Adaptation, IV, Pre-Exposure Tolerance of the Dark Adapted Peripheral Retina", Journal of the Optical Society of America, Volume 50, Number 6, June 1960, pp. 559 to 561.

Bhatia, Balraj, "Some Factors Determining the Maximum Ocular Movements", Journal of the Optical Society of America, Volume 50, Number 2, February 1960, pp. 149 to 150.

Plass, Gilbert, N., "A Method for Determination of Atmospheric Transmission Functions from Laboratory Absorption Measurements", Journal of the Optical Society of America, Volume 42, Number 9, September 1952, pp. 673 to 683.

Blackwell, Richard, "Studies of the Form of Visual Threshold Data", Journal of the Optical Society of America, Volume 43, Number 6, June 1953, pp. 456 to 463.

Stewart, Harold S., and Curcio, Joseph A., "The Influence of Field of View on Measurements of Atmospheric Transmission", Journal of the Optical Society of America, Volume 42, Number 11, November 1952, pp. 801 to 805.

Languet - Higgins, M. S., "Reflection and Refraction at a Random Moving Surface. I. Pattern and Paths of Specular Points", Journal of the Optical Society of America, Volume 50, Number 9, September 1960, pp. 838 to 844.

Peckman, R.H., "Visual Acuity, Contrast and Flicker, as Measures of Retinal Sensitivity", Journal of the Optical Society of America, Volume 42, Number 9, September 1952, pp. 621 to 625.

Vos, J. J., et al., "Visual Contrast Thresholds in Practical Problems", Journal of the Optical Society of America, Volume 46, Number 12, December 1956, pp. 1065 to 1070.

APPENDIX

The formula given by equation (10) gives the off-axis visual angle, ξ , within which 50 percent of the detections occur. It is thus an average angle giving the field of view for given aircraft and meteorological parameters. It has been assumed in the main body of this report that all points within the off-axis angle have a weighting value of unity and all points outside this angular region a weighting value of zero.

A better approximation to the exact value can be obtained by assuming an off-axis weighting distribution which is Gaussian with the average RMS angle defined in terms of ξ

where

$$\xi = \left[\frac{(3.0625 + 182.4\, C_o(R^2/A_p)\, e^{-3.44R/V})^{\frac{1}{2}} - 1.75}{91.2(R^2/A_p)}\right]^2.$$

which is a repetition of equation (10).

Then, the probability of detection as a function of the angular position and range of the aircraft relative to the line of sight can be expressed by the equation

$$P(\vartheta, R) = e^{-\frac{1}{2}(R\tan\vartheta/R\tan 1.43\xi)^2} \tag{60}$$

where

$P(\vartheta, R)$ is the probability of receiving enough light energy at the retina of the eye for perception as a function of off-axis angle and range.

If distant targets are considered, then ξ and ϑ (units in degrees) in general are small and the equation reduces to

$$P(\vartheta, R) = e^{-\frac{1}{2}(\vartheta/1.43\xi)^2}$$

The off-axis inefficiencies, $P(\vartheta, R)$, can be determined experimentally to see how accurately a Gaussian distribution describes the experimental results.

This can be done by using a stationary target with a fixed effective contrast to maintain the chance effect of seeing, P_{see}, constant. Then by many trials, at different off-axis positions, the number of detections made to the total number of trials at that off-axis position would give the probability of detection for that angular position. This can be repeated for all off-axis positions to check how closely the experimental results conform to the equation.

By including the sophistication for off-axis angle given in this appendix, the probability formulation for visual detection becomes

$$P(D/L_s) = P\left[\vartheta, d\vartheta/dt, R\right] = P(\vartheta, R) \cdot P_{see}(d\vartheta/dt, R) \qquad (62)$$

Since range, R, is very insensitive over one interval of the histogram, if the intervals chosen are small enough, the above probabilities in that interval are approximately independent and hence can be multiplied together.

One then obtains the wanted formulation by using equations (22), (10) and (61)

$$P(D/L_s) = P(\vartheta, d\vartheta/dt, R)$$

$$= \left\{1 - \left[1 - \frac{(k_1 C_{eff})^n e^{-k_1 C_{eff}}}{n!}\right]^{10}\right\} e^{-\frac{1}{2}(\vartheta/1.43\xi)^2}$$

where

$$C_{eff} = C_{eff}(d\vartheta/dt, R)$$

$$= C_o e^{-3.44R/V} \left(\frac{11}{d\vartheta/dt}\right)^{3/4} \cdot \frac{\tau_f}{0.21 + \tau_f} \qquad (63)$$

when $d\vartheta/dt > 11$ minutes/second

and

$$C_{eff} = C_o e^{-3.44R/V} \cdot \frac{\tau_f}{0.21 + \tau_f} \quad \text{when } d\vartheta/dt < 11 \text{ minutes/second}$$

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